



AFRL-AFOSR-VA-TR-2015-0282

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**Complexity, Robustness, and Multistability in Network Systems with Switching Topologies A Hierarchical Hybrid Control Approach**

Wassim Haddad  
GEORGIA TECH RESEARCH CORPORATION

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06/01/2015  
Final Report

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# REPORT DOCUMENTATION PAGE

*Form Approved  
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1. REPORT DATE (DD-MM-YYYY) <b>22-05-215</b>		2. REPORT TYPE <b>Final Report</b>		3. DATES COVERED (From - To) <b>May 1, 2011 – May 1, 2015</b>	
4. TITLE AND SUBTITLE  <b>Complexity, Robustness, and Multistability in Network Systems with Switching Topologies: A Hierarchical Hybrid Control Approach</b>				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER <b>FA9550-12-1-0192</b>	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S)  <b>Wassim M. Haddad</b>				5d. PROJECT NUMBER	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)  School of Aerospace Engineering Georgia Institute of Technology Atlanta, GA 30332-0150				8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)  Air Force Office of Scientific Research 875 North Randolph Street Suite 325, Room 3112 Arlington, VA 22203				10. SPONSOR/MONITOR'S ACRONYM(S)  Dynamics and Control	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION / AVAILABILITY STATEMENT  DISTRIBUTION A					
13. SUPPLEMENTARY NOTES					
14. ABSTRACT  Controls research under this program concentrated on the development of a unified discontinuous dynamical framework for nonlinear network systems. In particular, control algorithms were developed to address agent interactions, cooperative and non-cooperative control, task assignments, and resource allocations. To realize these tasks, appropriate sensory and cognitive capabilities such as adaptation, learning, decision-making, and agreement (or consensus) on the agent and multiagent levels were also developed. In addition, disturbance rejection and robustness for addressing communication/sensor noise and model uncertainties, as well as synchronism, system time-delays, and switching network topologies for addressing information asynchrony between agents, message transmission and processing delays, and communication link failures and communication dropouts were also addressed.					
15. SUBJECT TERMS Hybrid systems, network systems, large-scale systems, adaptive control, discontinuous systems					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES  <b>25</b>	19a. NAME OF RESPONSIBLE PERSON
a. REPORT Unclassified	b. ABSTRACT Unclassified	c. THIS PAGE Unclassified			19b. TELEPHONE NUMBER (include area code)

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**Final Report  
FA9550-12-1-0192**

by

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# REPORT DOCUMENTATION PAGE

*Form Approved  
OMB No. 0704-0188*

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# 1. Introduction

## 1.1. Research Objectives

As part of this research program we proposed to develop a unified discontinuous dynamical framework for nonlinear network systems. In particular, we concentrate on hybrid and hierarchical control algorithms to address agent interactions, cooperative and non-cooperative control, task assignments, and resource allocations. To realize these tasks, appropriate sensory and cogitative capabilities such as adaptation, learning, decision-making, and agreement (or consensus) on the agent and multiagent levels are developed. Application areas include spacecraft stabilization, cooperative control of unmanned air vehicles, network systems, and swarms of air and space vehicle formations.

## 1.2. Overview of Research

Due to advances in embedded computational resources over the last several years, a considerable research effort has been devoted to the control of networks and control over networks. Network systems involve distributed decision-making for coordination of networks of dynamic agents involving information flow enabling enhanced operational effectiveness via cooperative control in autonomous systems. These dynamical network systems cover a very broad spectrum of applications including cooperative control of unmanned air vehicles (UAV's) and autonomous underwater vehicles (AUV's) for combat, surveillance, and reconnaissance; distributed reconfigurable sensor networks for managing power levels of wireless networks; air and ground transportation systems for air traffic control and payload transport and traffic management; swarms of air and space vehicle formations for command and control between heterogeneous air and space vehicles; and congestion control in communication networks for routing the flow of information through a network.

As part of our research, over the last three years [1–44] we developed formation control protocols using a general control framework using hybrid stabilization of sets. The proposed framework develops a novel class of fixed-order, energy-based hybrid controllers as a means for achieving cooperative control formations, which can include flocking, cyclic pursuit, rendezvous, and consensus control of multiagent systems. These dynamic controllers combine a logical switching architecture with the continuous system dynamics to guarantee that a system generalized energy function whose zero level set characterizes a specified system formation is strictly decreasing across switchings. In addition, we developed sufficient conditions for gain, sector, and disk margins guarantees for Filippov nonlinear dynamical

systems controlled by optimal and inverse optimal discontinuous regulators. These results were used to design protocol controllers for group coordination of multiagent systems possessing a dynamic (i.e., switching) topology. Furthermore, we generalized the concepts of energy, entropy, and temperature to undirected and directed networks in order to demonstrate how thermodynamic principles can be applied to the design of distributed consensus control algorithms for networked dynamical systems.

We also developed stability, dissipativity, and optimality notions for dynamical systems with discontinuous vector fields. Specifically, we extended classical dissipativity theory to address the problem of dissipative discontinuous dynamical systems. Moreover, we considered discontinuous control problems involving a notion of optimality that is directly related to a specified nonsmooth Lyapunov function to obtain a characterization of optimal discontinuous feedback controllers. In addition, we developed a new consensus protocol for network multiagent systems using a resetting control architecture. Specifically, the control protocol consists of a delayed feedback, quasi-resetting control law such that controller resettings occur when the relative state measurements (i.e., distance) between an agent and its neighboring agents approach zero. Finally, we addressed the consensus problem for multiagent systems with uncertain interagent communication, wherein measurement uncertainty is characterized by balls of radius  $r$  centered at the neighboring agents exact locations. In particular, we show that the agents reach an almost consensus state and converge to a time-varying ball of radius  $r$  and include an analysis approach to the problem based on set-valued analysis.

### **1.3. Goals of this Report**

The main goal of this report is to summarize the progress achieved under the program during the past three years. Since most of the technical results appeared or will soon appear in over 40 archival journal and conference publications, we shall only summarize these results and remark on their significance and interrelationship.

## **2. Description of Work Accomplished**

The following partial research accomplishments have been completed over the past three years.

## 2.1. A Variational Approach to the Fuel Optimal Control Problem for UAV Formations

The pivotal role of unmanned aerial vehicles (UAVs) in modern aircraft technology is evidenced by the large number of civil and military applications they are employed in. For example, UAVs successfully serve as platforms carrying payloads aimed at land monitoring, wildfire detection and management, law enforcement, pollution monitoring, and communication broadcast relay, to name just a few examples.

A formation of UAVs, defined by a set of vehicles whose states are coupled through a common control law, is often more valuable than a single aircraft because it can accomplish several tasks concurrently. In particular, UAV formations can guarantee higher flexibility and redundancy, as well as increased capability of distributed payloads. For example, an aircraft formation can successfully intercept a vehicle which is faster than its chasers. Alternatively, a UAV formation equipped with interferometric synthetic aperture radar (In-SAR) antennas can pursue both along-track and cross-track interferometry, which allow harvesting information that a single radar cannot detect otherwise.

Path planning is one of the main problems when designing missions involving multiple vehicles; a UAV formation typically needs to accomplish diverse tasks while meeting some assigned constraints. For example, a UAV formation may need to intercept given targets while its members maintain an assigned relative attitude. Trajectories should also be optimized with respect to some performance measure capturing minimum time or minimum fuel expenditure. In particular, trajectory optimization is critical for mini and micro UAVs ( $\mu$ UAVs) because they often operate independently from remote human controllers for extended periods of time and also because of limited amount of available energy sources.

In this research [1], we provide a rigorous and sufficiently broad formulation of the optimal path planning problem for UAV formations, modeled as a system of  $n$  6-degrees of freedom (DoF) rigid bodies subject to a constant gravitational acceleration and aerodynamic forces and moments. Specifically, system trajectories are optimized in terms of control effort, that is, we design a control law that minimizes the forces and moments needed to operate a UAV formation, while meeting all the mission objectives. Minimizing the control effort is equivalent to minimizing the formation's fuel consumption in the case of vehicles equipped with conventional fuel-based propulsion systems and is a suitable indicator of the energy consumption for vehicles powered by batteries or other power sources.

In this research [1], we also derive an optimal control law which is independent of the size of the formation, the system constraints, and the environmental model adopted, and hence, our framework applies to aircraft, spacecraft, autonomous marine vehicles, and robot

formations. The direction and magnitude of the optimal control forces and moments is a function of the dynamics of two vectors, namely the translational and rotational primer vectors. In general, finding the dynamics of these two vectors over a given time interval is a demanding task that does not allow for an analytical closed-form solution, and hence, a numerical approach is required. Our main result involves necessary conditions for optimality of the formations' trajectories.

## 2.2. Output Feedback Adaptive Stabilization and Command Following for Minimum Phase Uncertain Dynamical Systems

There has been a number of results in recent decades focused on output feedback direct adaptive controllers. These results require an observer for unknown state variables, an observer for output tracking errors, an output predictor, and/or estimation of Markov parameters that lead to adaptive control algorithms with varying sets of assumptions. These assumptions include knowledge of the relative degree of the regulated system output and the dimension of the system, as well as the requirement that the system be minimum phase or passive. The main reason for the minimum phase assumption is because direct adaptive controllers employ high gain feedback that can drive nonminimum phase systems to instability.

In this research [3], we develop an output feedback adaptive control framework for continuous-time minimum phase multivariable uncertain dynamical systems with exogenous disturbances for output stabilization and command following. The approach is based on a nonminimal state space realization that generates an expanded set of states using the filtered inputs and filtered outputs, and their derivatives, of the original system. Specifically, a direct adaptive controller for the nonminimal state space model is constructed using the expanded states of the nonminimal realization and is shown to be effective for multi-input, multi-output linear dynamical systems with unmatched disturbances, unmatched uncertainties, and unstable dynamics. The proposed adaptive control architecture requires knowledge of the open-loop system's relative degree as well as a bound on the system's order. Several illustrative numerical examples are provided to demonstrate the efficacy of the proposed approach.

### 2.3. Nonlinear Differential Equations with Discontinuous Right-Hand Sides: Filippov Solutions, Nonsmooth Stability and Dissipativity Theory, and Optimal Discontinuous Feedback Control

Numerous engineering applications give rise to discontinuous dynamical systems. Specifically, in impact mechanics the motion of a dynamical system is subject to velocity jumps and force discontinuities leading to nonsmooth dynamical systems. In mechanical systems subject to unilateral constraints on system positions, discontinuities occur naturally through system-environment interactions. Alternatively, control of networks and control over networks with dynamic topologies also give rise to discontinuous systems. Specifically, link failures or creations in network systems result in switchings of the communication topology leading to dynamical systems with discontinuous right-hand sides. In addition, open-loop and feedback controllers also give rise to discontinuous dynamical systems. In particular, bang-bang controllers discontinuously switch between maximum and minimum control input values to generate minimum-time system trajectories, whereas sliding mode controllers use discontinuous feedback control for system stabilization. In switched systems, switching algorithms are used to select an appropriate plant (or controller) from a given finite parameterized family of plants (or controllers) giving rise to discontinuous systems.

In the case where the vector field defining the dynamical system is a discontinuous function of the state, system stability can be analyzed using nonsmooth Lyapunov theory involving concepts such as weak and strong stability notions, differential inclusions, and generalized gradients of locally Lipschitz continuous functions and proximal subdifferentials of lower semicontinuous functions. The consideration of nonsmooth Lyapunov functions for proving stability of discontinuous systems is an important extension to classical stability theory since there exist nonsmooth dynamical systems whose equilibria cannot be proved to be stable using standard continuously differentiable Lyapunov function theory.

In many applications of discontinuous dynamical systems such as mechanical systems having rigid-body modes, isospectral matrix dynamical systems, and consensus protocols for dynamical networks, the system dynamics give rise to a continuum of equilibria. Under such dynamics, the limiting system state achieved is not determined completely by the dynamics, but depends on the initial system state as well. For such systems possessing a continuum of equilibria, *semistability* [45], and not asymptotic stability, is the relevant notion of stability. Semistability is the property whereby every trajectory that starts in a neighborhood of a Lyapunov stable equilibrium converges to a (possibly different) Lyapunov stable equilibrium.

To address the stability analysis of discontinuous dynamical systems having a continuum

of equilibria, in this research [5, 12] we extend the theory of semistability to discontinuous time-invariant dynamical systems. In particular, we develop sufficient conditions to guarantee weak and strong invariance of Filippov solutions. Moreover, we present Lyapunov-based tests for semistability of autonomous differential inclusions. In addition, we develop sufficient conditions for finite-time semistability of autonomous discontinuous dynamical systems.

Many physical and engineering systems are *open systems*, that is, the system behaviour is described by an evolution law that involves the system state and the system input with, possibly, an output equation wherein past trajectories together with the knowledge of any inputs define future trajectories (uniquely or nonuniquely) and the system output depends on the instantaneous (present) values of the system state. Dissipativity theory is a system-theoretic concept that provides a powerful framework for the analysis and control design of open dynamical systems based on generalized system energy considerations. In particular, dissipativity theory exploits the notion that numerous physical dynamical systems have certain input-output and state properties related to conservation, dissipation, and transport of mass and energy. Such conservation laws are prevalent in dynamical systems, in general, and feedback control systems, in particular. The dissipation hypothesis on dynamical systems results in a fundamental constraint on the system dynamical behavior, wherein the stored energy of a dissipative dynamical system is at most equal to sum of the initial energy stored in the system and the total externally supplied energy to the system. Thus, the energy that can be extracted from the system through its input-output ports is less than or equal to the initial energy stored in the system, and hence, there can be no internal creation of energy; only conservation or dissipation of energy is possible.

The key foundation in developing dissipativity theory for nonlinear dynamical systems with continuously differentiable flows was presented by Willems in his seminal two-part paper on dissipative dynamical systems. In particular, Willems introduced the definition of dissipativity for general nonlinear dynamical systems in terms of a *dissipation inequality* involving a generalized system power input, or *supply rate*, and a generalized energy function, or *storage function*. The dissipation inequality implies that the increase in generalized system energy over a given time interval cannot exceed the generalized energy supply delivered to the system during this time interval. The set of all possible system storage functions is convex and every system storage function is bounded from below by the *available system storage* and bounded from above by the *required energy supply*.

In light of the fact that energy notions involving conservation, dissipation, and transport also arise naturally for discontinuous systems, it seems natural that dissipativity theory can play a key role in the analysis and control design of discontinuous dynamical systems.

Specifically, as in the analysis of continuous dynamical systems with continuously differentiable flows, dissipativity theory for discontinuous dynamical systems can involve conditions on system parameters that render an input, state, and output system dissipative. In addition, robust stability for discontinuous dynamical systems can be analyzed by viewing a discontinuous dynamical system as an interconnection of discontinuous dissipative dynamical subsystems. Alternatively, discontinuous dissipativity theory can be used to design discontinuous feedback controllers that add dissipation and guarantee stability robustness allowing discontinuous stabilization to be understood in physical terms. As for dynamical systems with continuously differentiable flows, dissipativity theory can play a fundamental role in addressing robustness, disturbance rejection, stability of feedback interconnections, and optimality for discontinuous dynamical systems.

In this research [5, 12], we develop Lyapunov-based tests for Lyapunov stability, semistability, finite-time stability, finite-time semistability, and asymptotic stability for nonlinear dynamical systems with discontinuous right-hand sides. Specifically, we develop new Lyapunov-based results for semistability that do not make assumptions of sign definiteness on the Lyapunov functions. Instead, our results use nontangency notions between the discontinuous vector field and weakly invariant or weakly negatively invariant subsets of the level or sublevel sets of the Lyapunov function. Moreover, using an extended notion of control Lyapunov functions we develop a universal feedback controller for discontinuous dynamical systems based on the existence of a nonsmooth control Lyapunov function defined in the sense of generalized Clarke gradients and set-valued Lie derivatives.

Next, we develop dissipativity notions for dynamical systems with discontinuous vector fields. Specifically, we consider dynamical systems with Lebesgue measurable and locally essentially bounded vector fields characterized by differential inclusions involving Filippov set-valued maps specifying a set of directions for the system velocity and admitting Filippov solutions with absolutely continuous curves. Moreover, we develop extended Kalman-Yakubovich-Popov conditions in terms of the discontinuous system dynamics for characterizing dissipativity via generalized Clarke gradients of locally Lipschitz continuous storage functions. In addition, using the concepts of dissipativity for discontinuous dynamical systems with appropriate storage functions and supply rates, we construct nonsmooth Lyapunov functions for discontinuous feedback systems by appropriately combining the storage functions for the forward and feedback subsystems. General stability criteria are given for Lyapunov, asymptotic, and exponential stability as well as finite-time stability for feedback interconnections of discontinuous dynamical systems. In the case where the supply rate involves the net system power or weighted input-output energy, these results provide extensions of the positivity and small gain theorems to discontinuous dynamical systems.

Finally, we consider a notion of optimality that is directly related to a given nonsmooth Lyapunov function. Specifically, an optimal control problem is stated and sufficient Hamilton-Jacobi-Bellman conditions are used to characterize an optimal discontinuous feedback controller. In addition, we develop sufficient conditions for gain, sector, and disk margin guarantees for Filippov nonlinear dynamical systems controlled by optimal and inverse optimal discontinuous regulators. Furthermore, we develop a counterpart to the classical return difference inequality for continuous-time systems with continuously differentiable flows for Filippov dynamical systems and provide connections between dissipativity and optimality for discontinuous nonlinear controllers. In particular, we show an equivalence between dissipativity and optimality of discontinuous controllers holds for Filippov dynamical systems. Specifically, we show that an optimal nonlinear controller  $\phi(x)$  satisfying a return difference condition is equivalent to the fact that the Filippov dynamical system with input  $u$  and output  $y = -\phi(x)$  is dissipative with respect to a supply rate of the form  $[u+y]^T[u+y] - u^T u$ .

## 2.4. Robust Adaptive Control Architecture for Disturbance Rejection and Uncertainty Suppression with $\mathcal{L}_\infty$ Transient and Steady-State Performance Guarantees

One of the fundamental problems in feedback control design is the ability of the control system to guarantee robust stability and robust performance with respect to system uncertainties in the design model. To this end, adaptive control along with robust control theory have been developed to address the problem of system uncertainty in control-system design. The fundamental differences between adaptive control design and robust control design can be traced to the modeling and treatment of system uncertainties as well as the controller architecture structures.

In particular, adaptive control is based on constant linearly parameterized system uncertainty models of a known structure but unknown variation, whereas robust control is predicated on structured and/or unstructured linear or nonlinear (possibly time-varying) operator uncertainty models consisting of bounded variation. Hence, for systems with constant real parametric uncertainties with large unknown variations, adaptive control is clearly appropriate, whereas for systems with time-varying parametric uncertainties and nonparametric uncertainties with norm bounded variations, robust control may be more suitable.

In contrast to fixed-gain robust controllers, which are predicated on a mathematical model of the system uncertainty and which maintain specified constants within the feedback control law to *sustain* robust stability and performance over the range of system uncertainty, adaptive controllers directly or indirectly adjust feedback gains to maintain closed-loop stability

and *improve* performance in the face of system uncertainties. Specifically, indirect adaptive controllers utilize parameter update laws to identify unknown system parameters and adjust feedback gains to account for system variation, whereas direct adaptive controllers directly adjust the controller gains in response to plant variation. In either case, the overall process of parameter identification and controller adjustment constitutes a nonlinear control law architecture, which makes validation and verification of guaranteed transient and steady-state performance, as well as robustness margins of adaptive controllers extremely challenging.

While adaptive control has been used in numerous applications to achieve system performance without excessive reliance on system models, the necessity of high-gain feedback for achieving fast adaptation can be a serious limitation of adaptive controllers. Specifically, in certain applications fast adaptation is required to achieve stringent tracking performance specifications in the face of large system uncertainties and abrupt changes in system dynamics. This, for example, is the case for high performance aircraft systems that can be subjected to system faults or structural damage which can result in major changes in aerodynamic system parameters. In such situations, high-gain adaptive control is necessary in order to rapidly reduce and maintain system tracking errors. However, fast adaptation using high-gain feedback can result in high-frequency oscillations which can excite unmodeled system dynamics resulting in system instability. Hence, there exists a critical trade-off between system stability and control adaptation rate.

Virtually all adaptive control methods developed in the literature have averted the problem of high-gain control. Notable exceptions include the use of a low-pass filter that effectively subverts high frequency oscillations that can occur due to fast adaptation while using a predictor model to reconstruct the reference system model. In particular, this method involves a robust adaptive control architecture that provides sufficient conditions for stability and performance in terms of  $\mathcal{L}_1$ -norms of the underlying system transfer functions despite fast adaptation, leading to uniform bounds on the  $\mathcal{L}_\infty$ -norms of the system input-output signals.

In this research [4], a new adaptive control architecture for linear and nonlinear uncertain dynamical systems is developed to address the problem of high-gain adaptive control. Specifically, the proposed framework involves a new and novel controller architecture involving a modification term in the update law that minimizes an error criterion involving the distance between the weighted regressor vector and the weighted system error states. This modification term allows for fast adaptation without hindering system robustness. In particular, we show that the governing tracking closed-loop system error equation approximates a Hurwitz linear time-invariant dynamical system with  $\mathcal{L}_\infty$  input-output signals. This key

feature of our framework allows for robust stability analysis of the proposed adaptive control law using  $\mathcal{L}_1$  system theory. Specifically, in the face of fast adaptation, uniform transient and steady-state system performance bounds are derived in terms of  $\mathcal{L}_1$ -norms of the closed-loop system error dynamics. We further show that by properly choosing the design parameters in the modification term we can adjust the bandwidth of the adaptive controller, the transient and steady-state closed-loop performance, and the size of the ultimate bound of the closed-loop system trajectories independently of the system adaptation rate. Several illustrative numerical examples are provided to demonstrate the efficacy of the proposed approach.

## 2.5. Formation Control Protocols for Nonlinear Dynamical Systems via Hybrid Stabilization of Sets

Using system-theoretic thermodynamic concepts, an energy- and entropy-based hybrid controller architecture was proposed in [46] as a means for achieving enhanced energy dissipation in lossless and dissipative dynamical systems. These dynamic controllers combined a logical switching architecture with continuous dynamics to guarantee that the system plant energy is strictly decreasing across switchings. The general framework developed in [46] leads to closed-loop systems described by impulsive differential equations [46]. In particular, the Principal Investigator and his collaborators in [46] construct hybrid dynamic controllers that guarantee that the closed-loop system is consistent with basic thermodynamic principles. Specifically, the existence of an entropy function for the closed-loop system is established that satisfies a hybrid Clausius-type inequality. Special cases of energy-based and entropy-based hybrid controllers involving state-dependent switching were also developed to show the efficacy of the approach.

Recent technological advances in communications and computation have spurred a broad interest in control of networks and control over networks. Network systems involve distributed decision-making for coordination of networks of dynamic agents and address a broad area of applications including cooperative control of unmanned air vehicles, microsatellite clusters, mobile robotics, and congestion control in communication networks. In many applications involving multiagent systems, groups of agents are required to agree on certain quantities of interest. For example, in a group of autonomous vehicles this property might be a common heading angle or a shared communication frequency. Moreover, it is important to develop information consensus protocols for networks of dynamic agents, wherein a unique feature of the closed-loop dynamics under any control algorithm that achieves consensus is the existence of a continuum of equilibria representing a state of equipartitioning or *consensus*. Under such dynamics, the limiting consensus state achieved is not determined

completely by the dynamics, but depends on the initial system state as well. For such systems possessing a continuum of equilibria, *semistability* [45], and not asymptotic stability, is the relevant notion of stability. In addition, system-theoretic thermodynamic concepts [47] have proved invaluable in addressing Lyapunov stability and convergence for nonlinear dynamical networks.

Convergence and state equipartitioning also arise in numerous complex large-scale dynamical networks that demonstrate a degree of synchronization. System synchronization typically involves coordination of events that allows a dynamical system to operate in unison resulting in system self-organization. The onset of synchronization in populations of coupled dynamical networks have been studied for various complex networks including network models for mathematical biology, statistical physics, kinetic theory, bifurcation theory, as well as plasma physics. Synchronization of firing neural oscillator populations also appears in the neuroscience literature.

Alternatively, in other applications of multiagent systems, groups of agents are required to achieve and maintain a prescribed geometric shape. This *formation control* problem includes *flocking* and *cyclic pursuit*, wherein parallel and circular formations of vehicles are sought. For formation control of multiple vehicles, *cohesion*, *separation*, and *alignment* constraints are typically required for individual agent steering which describe how a given vehicle maneuvers based on the positions and velocities of nearby agents. Specifically, cohesion refers to a steering rule wherein a given vehicle attempts to move toward the average position of local vehicles, separation refers to collision avoidance with nearby vehicles, whereas alignment refers to velocity matching with nearby vehicles.

Since a specified formation of multiagent systems, which can include flocking, cyclic pursuit, rendezvous, or consensus, can be characterized by a hyperplane or manifold in the state space, in this research [13] we extend the results of [46] to develop a state-dependent hybrid control framework for addressing multiagent formation control protocols for general nonlinear dynamical systems using hybrid stabilization of sets. The proposed framework involves a novel class of fixed-order, energy-based hybrid controllers as a means for achieving cooperative control formations. These dynamic controllers combine a logical switching architecture with continuous dynamics to guarantee that a system generalized energy function, whose zero level set characterizes a specified system formation, is strictly decreasing across switchings. The general framework leads to hybrid closed-loop systems described by impulsive differential equations and addresses general nonlinear dynamical systems without limiting consensus and formation control protocols to single and double integrator models.

## 2.6. On the Equivalence Between Dissipativity and Optimality of Discontinuous Nonlinear Regulators for Filippov Dynamical Systems

For continuous-time nonlinear dynamical systems with continuously differentiable flows, the problem of guaranteed stability margins for optimal and inverse optimal regulators is well known [45]. Specifically, nonlinear inverse optimal controllers that minimize a *meaningful* nonlinear-nonquadratic performance criterion involving a nonlinear-nonquadratic, nonnegative-definite function of the state and a quadratic positive definite function of the control are known to possess sector margin guarantees to component decoupled memoryless input nonlinearities lying in the conic sector  $(\frac{1}{2}, \infty)$ . These results also hold for disk margin guarantees where asymptotic stability of the closed-loop system is guaranteed in the face of a dissipative dynamic input operator. In addition, using a certain return difference condition, closely related to loop gain concepts in linear systems theory, an equivalence between dissipativity with respect to a quadratic supply rate and optimality of a nonlinear feedback regulator also holds.

In [46], the Principal Investigator extended the results of [45] to develop a general framework for hybrid feedback systems by addressing stability, dissipativity, optimality, and inverse optimality of impulsive dynamical systems. In particular, [46] considers a hybrid feedback optimal control problem over an infinite horizon involving a hybrid nonlinear-nonquadratic performance functional. In addition, sufficient conditions for hybrid gain, sector, and disk margins guarantees for nonlinear hybrid dynamical systems were developed.

In recent research [5], we developed input-output and state dissipativity notions for dynamical systems with discontinuous vector fields. Specifically, we consider dynamical systems with Lebesgue measurable and locally essentially bounded vector fields characterized by differential inclusions involving set-valued maps specifying a set of directions for the system velocity and admitting solutions with absolutely continuous curves. In particular, we introduce a generalized definition of dissipativity for discontinuous dynamical systems in terms of set-valued supply rate maps and set-valued storage maps consisting of locally Lebesgue integrable supply rates and Lipschitz continuous storage functions, respectively. In addition, we introduce the notion of a *set-valued available storage map* and a *set-valued required supply rate map*, and show that if these maps have closed convex images they specialize to single-valued maps corresponding to the *smallest available storage* and the *largest required supply* of the differential inclusion, respectively. Furthermore, we show that all system storage functions are bounded from above by the largest required supply and bounded from below by the smallest available storage, and hence, as in the case for systems with continuously

differentiable flows, a dissipative differential inclusion can deliver to its surroundings only a fraction of its generalized stored energy and can store only a fraction of the generalized work done to it.

In this research [9], we use the results of [12] to develop extended Kalman–Yakubovich–Popov conditions, in terms of the discontinuous system dynamics, characterizing dissipativity via generalized Clarke gradients and locally Lipschitz continuous storage functions. In addition, we develop sufficient conditions for gain, sector, and disk margins guarantees for Filippov nonlinear dynamical systems controlled by optimal and inverse optimal discontinuous regulators. Furthermore, we develop a counterpart to the classical return difference inequality for continuous-time systems with continuously differentiable flows for Filippov dynamical systems and provide connections between dissipativity and optimality for discontinuous nonlinear controllers. In particular, we show an equivalence between dissipativity and optimality of discontinuous controllers holds for Filippov dynamical systems. Specifically, we show that an optimal nonlinear controller  $\phi(x)$  satisfying a return difference condition is equivalent to the fact that the Filippov dynamical system with input  $u$  and output  $y = -\phi(x)$  is dissipative with respect to a supply rate of the form  $[u + y]^T[u + y] - u^T u$ .

## 2.7. Thermodynamics-Based Control of Network Systems

For a network of interconnected dynamical systems it is often desired that some property of each subsystem approaches a single common value across the network. For example, in a group of autonomous vehicles this property might be a common heading angle or a shared communication frequency. Designing a controller that ensures that a common value will be found is called the *consensus control problem*. Achieving consensus with distributed controllers that can access only local information is called the *distributed consensus control problem*. Related topics include rendezvous, synchronization, flocking, and cyclic pursuit. As noted in Section 2.5, these topics arise in a broad variety of important applications, including cooperative control of unmanned air vehicles, microsatellite clusters, mobile robots, and congestion control in communication networks.

A sizable body of work has emerged in recent years that addresses the distributed consensus problem using the tools of algebraic graph theory. In this research [8], we present an alternative perspective to the distributed consensus problem, based on *system thermodynamics*, a framework that unifies the foundational disciplines of thermodynamics and dynamical system theory [47]. System thermodynamics has been applied to achieve the formulation of classical thermodynamics in a dynamical systems setting [47]. System thermodynamics has also been used to apply thermodynamic principles to the analysis, design, and control

of dynamic systems [46].

To illustrate the relevance of thermodynamics to consensus control, consider conductive heat flow in a homogeneous, isotropic, thermally insulated body. Heat flows within the body according to Fourier’s law of heat conduction, which states that the rate of heat flow  $q$  through an area  $A$  is proportional to the temperature gradient  $\nabla T$ . The constant of proportionality is an intrinsic material property called the *thermal conductivity* and denoted by  $\kappa$ . Hence,  $q = -\kappa A \nabla T$ . One consequence of the second law of thermodynamics is that  $\kappa$  cannot be negative; another is that heat will flow until the temperature of the body is uniform. If the system is perturbed by local addition or removal of heat, then the temperature distribution will respond by stabilizing at a new uniform value. That is, the natural flow of heat under Fourier’s law robustly maintains a globally stable “temperature consensus” in response to system disturbances and system uncertainty. Just as intuitive notions of energy and dissipation can guide controller design using Lyapunov or passivity-based methods [45], so can the laws of thermodynamics be abstracted and generalized to guide controller design for consensus control problems in networked systems [8].

In this research [8], we develop a system thermodynamic framework for the distributed consensus control problem on static, finite-dimensional, undirected and directed networks. We show that system thermodynamics proves extremely effective in designing controllers for such systems, and the intuition provided by the thermodynamic analogies points the way towards extensions to a broader class of problems.

## 2.8. Semistabilization, Feedback Dissipativity, System Thermodynamics, and Limits of Performance in Feedback Control

In this research [28], we develop a thermodynamic framework for semistabilization of linear and nonlinear dynamical systems. The proposed framework unifies system thermodynamic concepts with feedback dissipativity and control theory to provide a thermodynamic-based semistabilization framework for feedback control design. Specifically, we consider feedback passive and dissipative systems [45] since these systems are not only widespread in system engineering, but also have clear connections to thermodynamics [47]. In addition, using ideas from [47], we define the notion of entropy for a nonlinear feedback dissipative dynamical system. Then, we develop a state feedback control design framework that minimizes the time-averaged system entropy and show that, under certain conditions, this controller also minimizes the time-averaged system energy. The main result is cast as an optimal control problem characterized by an optimization problem involving two linear matrix inequalities.

In future research, we plan to merge the system thermodynamic semistabilization frame-

work involving the singular control performance criterion considered in [20] and the feedback limitation framework for nonlinear dynamical systems using Bode integrals and cheap control to develop a unified nonlinear stabilization framework with a priori achievable system performance guarantees.

## 2.9. Adaptive Estimation using Multiagent Network Identifiers with Undirected and Directed Graph Topologies

In this research [10], we consider the problem of adaptive estimation of a linear system with unknown plant and input matrices. In particular, we propose a novel distributed observer architecture that adaptively identifies the dynamic system matrices using a group of  $n$  agents. Each agent generates its own adaptive identifier which is based on a particular identifier architecture. Furthermore, it is shown that if the adaptive identifiers have the same structure, but do not share information (i.e., are not connected), then there is no guarantee that the  $n$  adaptive identifiers will have their estimates converge to the same value without a persistency of excitation condition being imposed. Alternatively, when the update laws for the parameter identifiers are modified to include interagent information exchange, then consensus of both the state and parameter estimates are guaranteed, and thus, emulating a persistency of excitation condition.

The proposed adaptive identifier architecture includes additional terms in both the state and parameter equations, which effectively penalize the mismatch between all estimates and take the form of nonnegative damping terms that serve to enhance the convergence properties of the state and parameter errors. The added benefit of the proposed network architecture of the adaptive identifiers, which penalize the mismatch between both state and parameter estimates, is the abstract form that the collective error dynamics take. In particular, the proposed framework allows one to decouple the graph connectivity (i.e., the graph Laplacian) from the stability analysis of the parameter errors by simply replacing a nonnegative damping-like matrix representing the connectivity of the graph topology with another matrix representing a more general interagent connectivity.

## 2.10. Consensus Protocols for Networked Multiagent Systems with a Uniformly Continuous Quasi-Resetting Architecture

Networked multiagent systems consists of a group of agents that locally sense their environment, communicate with each other, and process information in order to achieve a given set of system objectives. Since these systems have widespread applications in physics, bi-

ology, social sciences, economics, and engineering, it is not surprising that the last decade has witnessed an increased interest in networked multiagent systems. For a multi-vehicle aerospace network of interconnected systems, it is often desired that some property of each vehicle approach a single common value across the network. For example, in a group of autonomous aerospace vehicles this property might be a common heading angle or a shared communication frequency. Designing a controller that ensures a set of system objectives is called the *consensus control* problem. Consensus control protocols employ a distributed controller architecture wherein local information is accessed and processed.

In this research [11], we present a novel network consensus control protocol using an approximate resetting architecture. Specifically, we develop continuous approximate resetting controllers for networked multiagent systems. This controller framework leads to a new consensus protocol architecture consisting of a delayed feedback control law with uniformly continuous quasi-resettings occurring when the relative state measurements (i.e., distance) between an agent and its neighboring agents approach zero. Furthermore, we show that the proposed framework does not require any well-posedness assumptions nor time regularization that is typically imposed by hybrid resetting controllers [46]. Using a Lyapunov-Krasovskii functional, we also show that the multiagent system reaches asymptotic agreement, wherein the system steady-state is uniformly distributed over the system initial conditions preserving the centroid of the network. In addition, we develop  $\mathcal{L}_\infty$  consensus performance guarantees while accounting for system overshoot constraints and excessive control effort.

## 2.11. Set-Valued Protocols for Almost Consensus of Multiagent Systems with Uncertain Interagent Communication

In this research [21,38], we consider a multiagent consensus problem in which agents have sensors with limited accuracy. Specifically, in numerous network system applications agents can detect the location of the neighboring agents only approximately. This problem arises in robotics applications involving low sensor quality or detrimental environmental conditions. In such a setting, it is desirable that the agents reach consensus approximately. We develop a set-valued consensus protocol that guarantees that the agents converge to a time-varying set of diameter  $2r$  when the agents have sensors that can detect the location of the neighboring agents with accuracy up to a ball of radius  $r$  centered at the actual location of the neighboring agents. This set is shown to be time-varying, in the sense that only the differences between agents positions are, in the limit, small. Due to the uncertainty in interagent communication, we use difference inclusions and set-valued analysis to describe the problem formulation.

In this research [21,38], we develop almost consensus protocols for multiagent systems

with uncertain interagent communication. Specifically, the proposed protocol algorithm modifies the set-valued consensus update maps of the agents by assuming that the locations of all agents, including the agents calculating the update map, are within a ball of radius  $r$ . However, since the update sets of our design protocol do not satisfy a strict convexity assumption, our results go beyond the results reported in the literature by employing a set-valued invariance principle.

## 2.12. A Universal Feedback Controller for Discontinuous Dynamical Systems using Nonsmooth Control Lyapunov Functions

The consideration of nonsmooth Lyapunov functions for proving stability of feedback discontinuous systems is an important extension to classical stability theory since there exist nonsmooth dynamical systems whose equilibria cannot be proved to be stable using standard continuously differentiable Lyapunov function theory. For dynamical systems with continuously differentiable flows, the concept of smooth control Lyapunov functions was developed by Artstein to show the existence of a feedback stabilizing controller. A constructive feedback control law based on smooth control Lyapunov functions was given by Sontag.

Even though a stabilizing continuous feedback controller guarantees the existence of a smooth control Lyapunov function, many systems that possess smooth control Lyapunov functions do not necessarily admit a continuous stabilizing feedback controller. However, the existence of a control Lyapunov function allows for the design of a stabilizing feedback controller that admits Filippov and Krasovskii closed-loop system solutions. Furthermore, the problem of stabilization of globally asymptotically controllable systems wherein the system vector field is locally Lipschitz continuous in the state and uniformly in the control has been addressed in the literature. For the aforementioned class of systems, researchers have constructed a discontinuous control law using semiconcave control Lyapunov functions in the sense of proximal subdifferentials. However, we do not need to consider semiconcavity in our work.

In this research [14], we build on the results of [5, 9, 12] to develop a constructive universal feedback control law for discontinuous dynamical systems based on the existence of a nonsmooth control Lyapunov function defined in the sense of generalized Clarke gradients and set-valued Lie derivatives [12]. Specifically, we address the problem of discontinuous stabilization for dynamical systems with Lebesgue measurable and locally essentially bounded vector fields characterized by differential inclusions involving Filippov set-valued maps and admitting Filippov solutions with absolutely continuous curves.

## 2.13. Optimal Control for Linear and Nonlinear Semistabilization

A form of stability that lies between Lyapunov stability and asymptotic stability is *semistability* [45], that is, the property whereby every trajectory that starts in a neighborhood of a Lyapunov stable equilibrium converges to a (possibly different) Lyapunov stable equilibrium. Semistability implies Lyapunov stability, and is implied by asymptotic stability. This notion of stability arises naturally in systems having a continuum of equilibria and includes such systems as mechanical systems having rigid body modes, chemical reaction systems, compartmental systems, and isospectral matrix dynamical systems. Semistability also arises naturally in dynamical network systems, which cover a broad spectrum of applications including cooperative control of unmanned air vehicles, autonomous underwater vehicles, distributed sensor networks, air and ground transportation systems, swarms of air and space vehicle formations, and congestion control in communication networks, to cite but a few examples.

A unique feature of the closed-loop dynamics under any control algorithm that achieves consensus in dynamic networks is the existence of a continuum of equilibria representing a desired state of consensus. Under such dynamics, the desired limiting state is not determined completely by the closed-loop system dynamics, but depends on the initial system state as well. From a practical viewpoint, it is not sufficient to only guarantee that a network converges to a state of consensus since steady-state convergence is not sufficient to guarantee that small perturbations from the limiting state will lead to only small transient excursions from the state of consensus. It is also necessary to guarantee that the equilibrium states representing consensus are Lyapunov stable, and consequentially, semistable.

In this research [15], we address the problem of finding a state-feedback nonlinear control law that minimizes a nonlinear-nonquadratic performance measure and guarantees semistability of a nonlinear dynamical system. Specifically, our approach focuses on the role of the Lyapunov function guaranteeing semistability of the closed-loop system and we provide sufficient conditions for optimality in a form that corresponds to a steady-state version of a Hamilton-Jacobi-Bellman-type equation.

## 2.14. Finite-Time Partial Stability and Stabilization, and Optimal Feedback Control

In [45] the current status of continuous-time, nonlinear nonquadratic optimal control problems was presented in a simplified and tutorial manner. The basic underlying ideas of the results in [45] are based on the fact that the steady-state solution of the Hamilton-

Jacobi-Bellman equation is a Lyapunov function for the nonlinear system and thus guaranteeing both stability and optimality. Specifically, a feedback control problem over an infinite horizon involving a nonlinear-nonquadratic performance functional is considered. The performance functional is then evaluated in closed form as long as the nonlinear nonquadratic cost functional considered is related in a specific way to an underlying Lyapunov function that guarantees asymptotic stability of the nonlinear closed-loop system. This Lyapunov function is shown to be the solution of the steady-state Hamilton-Jacobi-Bellman equation. The overall framework provides the foundation for extending linear-quadratic control to nonlinear-nonquadratic problems.

In this research [16], we extend the framework developed in [45] to address the problem of optimal *finite-time stabilization*, that is, the problem of finding state-feedback control laws that minimize a given performance measure and guarantee finite-time stability of the closed-loop system. In addition, we address the problem of optimal *partial-state stabilization*, wherein stabilization with respect to a subset of the system state variables is desired.

Specifically, we consider a notion of optimality that is directly related to a given Lyapunov function that is positive definite and decrescent with respect to part of the system state, and satisfies a differential inequality involving fractional powers. In particular, an *optimal* finite-time, partial-state stabilization control problem is stated and sufficient Hamilton-Jacobi-Bellman conditions are used to characterize an optimal feedback controller. The steady-state solution of the Hamilton-Jacobi-Bellman equation is clearly shown to be a Lyapunov function for part of the closed-loop system state that guarantees both finite-time partial stability and optimality. In addition, we explore connections of our approach with inverse optimal control, wherein we parametrize a family of finite-time, partial-state stabilizing sublinear controllers that minimize a derived cost functional involving subquadratic terms. Another important application of partial stability and partial stabilization theory is the unification it provides between time-invariant stability theory and stability theory for time-varying systems [45]. We exploit this unification and specialize our results to address the problem of optimal finite-time control for nonlinear time-varying dynamical systems.

## 2.15. Partial-State Stabilization and Optimal Feedback Control

In this research [17], we extend the framework developed in [45] to address the problem of optimal *partial-state stabilization*, wherein stabilization with respect to a subset of the system state variables is desired. Partial-state stabilization arises in many engineering applications. Specifically, in spacecraft stabilization via gimballed gyroscopes asymptotic stability of an equilibrium position of the spacecraft is sought while requiring Lyapunov

stability of the axis of the gyroscope relative to the spacecraft . Alternatively, in the control of rotating machinery with mass imbalance, spin stabilization about a nonprincipal axis of inertia requires motion stabilization with respect to a subspace instead of the origin. Perhaps the most common application where partial stabilization is necessary is adaptive control, wherein asymptotic stability of the closed-loop plant states is guaranteed without necessarily achieving parameter error convergence. The need to consider partial stability of the closed-loop system in the aforementioned systems arises from the fact that stability notions involve equilibrium coordinates as well as a manifold of coordinates that is closed but *not* compact. Hence, partial stability involves motion lying in a subspace instead of an equilibrium point.

Even though partial-state stabilization has been considered in the literature, the problem of optimal partial-state stabilization has received very little attention. In this research [17], we consider a notion of optimality that is directly related to a given Lyapunov function that is positive definite and decrescent with respect to part of the system state. Specifically, an optimal partial-state stabilization control problem is stated and sufficient Hamilton-Jacobi-Bellman conditions are used to characterize an optimal feedback controller. Another important application of partial stability and partial stabilization theory is the unification it provides between time-invariant stability theory and stability theory for time-varying systems [45]. We exploit this unification and specialize our results to address optimal linear and nonlinear regulation for linear and nonlinear time-varying systems with quadratic and nonlinear nonquadratic cost functionals.

### **3. Research Personnel Supported**

#### **Faculty**

Wassim M. Haddad, Principal Investigator

#### **Graduate Students**

T. Sadikhov, Ph. D, and A. L’Afflitto, Ph. D.

One other student (T. Rajpurhoit) was involved in research projects that were closely related to this program. Although he was not financially supported by this program, his research did directly contribute to the overall research effort. Furthermore, two Ph. D. dissertations were completed under partial support of this program; namely

A. L’Afflitto, *Finite-Time Partial Stability, Stabilization, Semistabilization, and Optimal Feedback Control*, April 2015.

T. Sadikhov, *Stability, Dissipativity, and Optimal Control of Discontinuous Dynamical Systems*, April 2015.

Dr. L’Afflitto has accepted a position as an Assistant Professor in the School of Aerospace and Mechanical Engineering at The University of Oklahoma, Norman, OK, and Dr. Sadikhov has accepted a position with Mercedes-Benz Research and Development, Autonomous Systems Division, in Sunnyvale, CA.

## 4. Interactions and Transitions

### 4.1. Participation and Presentations

The following conferences were attended over the past three years.

American Control Conference, Montreal, Canada, June 2012.

IEEE Conference on Decision and Control, Maui, HI, December 2012.

American Control Conference, Washington, DC, June 2013.

IEEE Mediterranean Conference on Control and Automation, Chania, Greece, June 2013.

IEEE Conference on Decision and Control, Florence, Italy, December 2013.

American Control Conference, Portland, OR, June 2014.

IEEE Conference on Decision and Control, Los Angeles, CA, December 2014.

Furthermore, conference articles [22-44] were presented.

### 4.2. Transitions

Our work on adaptive and neuroadaptive control of drug delivery partially supported under this and previous AFSOR programs continues to transition to clinical studies at the Northeast Georgia Medical Center in Gainesville, Georgia, under the direction of Dr. James M. Bailey (770-534-1312), director of cardiac anesthesia and consultant in critical care medicine. This work has recently transitioned from operating room (OR) hypnosis to intensive care unit (ICU) sedation. In addition, this work was communicated to Colonel Leopoldo C. Cancio (210-916-3301) of the US Army Institute of Surgical Research in Fort Sam Houston, San Antonio, in order to provide improvements for combat casualty care in current and future battlefields. Collaboration with the US Army Institute of Surgical Research is underway.

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1.

**1. Report Type**

Final Report

**Primary Contact E-mail**

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**Primary Contact Phone Number**

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404-894-1078

**Organization / Institution name**

Georgia Institute of Technology

**Grant/Contract Title**

**The full title of the funded effort.**

Complexity, Robustness, and Multistability in Network Systems with Switching Topologies: A Hierarchical Hybrid Control Approach

**Grant/Contract Number**

**AFOSR assigned control number. It must begin with "FA9550" or "F49620" or "FA2386".**

FA9550-12-1-0192

**Principal Investigator Name**

**The full name of the principal investigator on the grant or contract.**

Wassim M. Haddad

**Program Manager**

**The AFOSR Program Manager currently assigned to the award**

Wassim M. Haddad

**Reporting Period Start Date**

05/01/2012

**Reporting Period End Date**

04/30/2015

**Abstract**

Controls research under this program concentrated on the development of a unified discontinuous dynamical framework for nonlinear network systems. In particular, control algorithms were developed to address agent interactions, cooperative and non-cooperative control, task assignments, and resource allocations. To realize these tasks, appropriate sensory and cognitive capabilities such as adaptation, learning, decision-making, and agreement (or consensus) on the agent and multiagent levels were also developed. In addition, disturbance rejection and robustness for addressing communication/sensor noise and model uncertainties, as well as synchronism, system time-delays, and switching network topologies for addressing information asynchrony between agents, message transmission and processing delays, and communication link failures and communication dropouts were also addressed.

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**LRIR Title**

**Reporting Period**

**Laboratory Task Manager**

**Program Officer**

**Research Objectives**

**Technical Summary**

**Funding Summary by Cost Category (by FY, \$K)**

	Starting FY	FY+1	FY+2
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Supplies			
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**2. Thank You**

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